Erratum: ”Estimates on periodic and Dirichlet eigenvalues for the Zakharov-Shabat system” [Asymptot. Anal. 25 (2001), no. 3-4, 201–237]

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Erratum

Estimates on periodic and Dirichlet eigenvalues for the Zakharov–Shabat system
by B. Grébert and T. Kappeler

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It was brought to our attention that inequality (A.6) on p. 232 in the proof of Proposition A.1 is wrong and therefore the proof for the bound (A.8) not correct.

The proof of Proposition A.1 can be easily corrected. Define for $n \geqslant 1$ the union of contours

$$\mathcal{R}_n = \left\{\lambda \in \mathbb{C} \mid |\lambda| = n\pi - \frac{\pi}{2}\right\} \cup \left(\bigcup_{|k| \geqslant n} \left\{\lambda \in \mathbb{C} \mid |\lambda - k\pi| = \frac{\pi}{2}\right\}\right).$$

For any $\lambda \in \bigcup_{n\geqslant 1} \mathcal{R}_n$, one has

$$L - \lambda = (\mathrm{Id} + Q_\lambda)(L_0 - \lambda),$$

where Q_λ is the bounded operator on L^2 given by

$$Q_\lambda = \begin{pmatrix} 0 & \psi_1 \\ \psi_2 & 0 \end{pmatrix} (L_0 - \lambda)^{-1}.$$

To prove that $L - \lambda$ is invertible for any given λ in $\bigcup_{n\geqslant 1} \mathcal{R}_n$ it suffices to show that $\mathrm{Id} + Q_\lambda$ is invertible. As in Section 2.2 we notice that whenever $\mathrm{Id} - Q_\lambda^2$ is invertible one has

$$\mathrm{Id} = (\mathrm{Id} + Q_\lambda) \circ (\mathrm{Id} - Q_\lambda) \circ \left(\mathrm{Id} - Q_\lambda^2\right)^{-1}, \tag{A.5*}$$

which implies that $\mathrm{Id} + Q_\lambda$ is invertible. Using the assumption that w is a δ-weight one concludes from Lemmas 2.1 and 2.2 that there exists $N \geqslant 1$ so that for any $\lambda \in \bigcup_{n\geqslant N} \mathcal{R}_n$ and any 1-periodic potentials $\psi_1, \psi_2 \in H^w$ with $\|\psi_j\|_w \leqslant M$ one has

$$\|Q_\lambda^2\|_{\mathcal{L}(L^2)} \leqslant \frac{1}{2}. \tag{A.8*}$$

The proof of Proposition A.1 can then be finished as in Appendix A and Lemma A.2 follows, using (A.5*) and (A.8*).